Statistics: Description and inference

Part II: the t-test

Statistical theory and process control

- Consider the problem facing Gossett or any quality control engineer. At a brewery (or any factory), beer (or widgets) are produced to meet certain specifications. There is a certain amount of variation from specifications that is acceptable, but you need to detect when something has gone wrong; i.e., when specifications are no longer being met. How can you tell if the product is being made up to specification?
- Two basic cases: Large samples and small samples

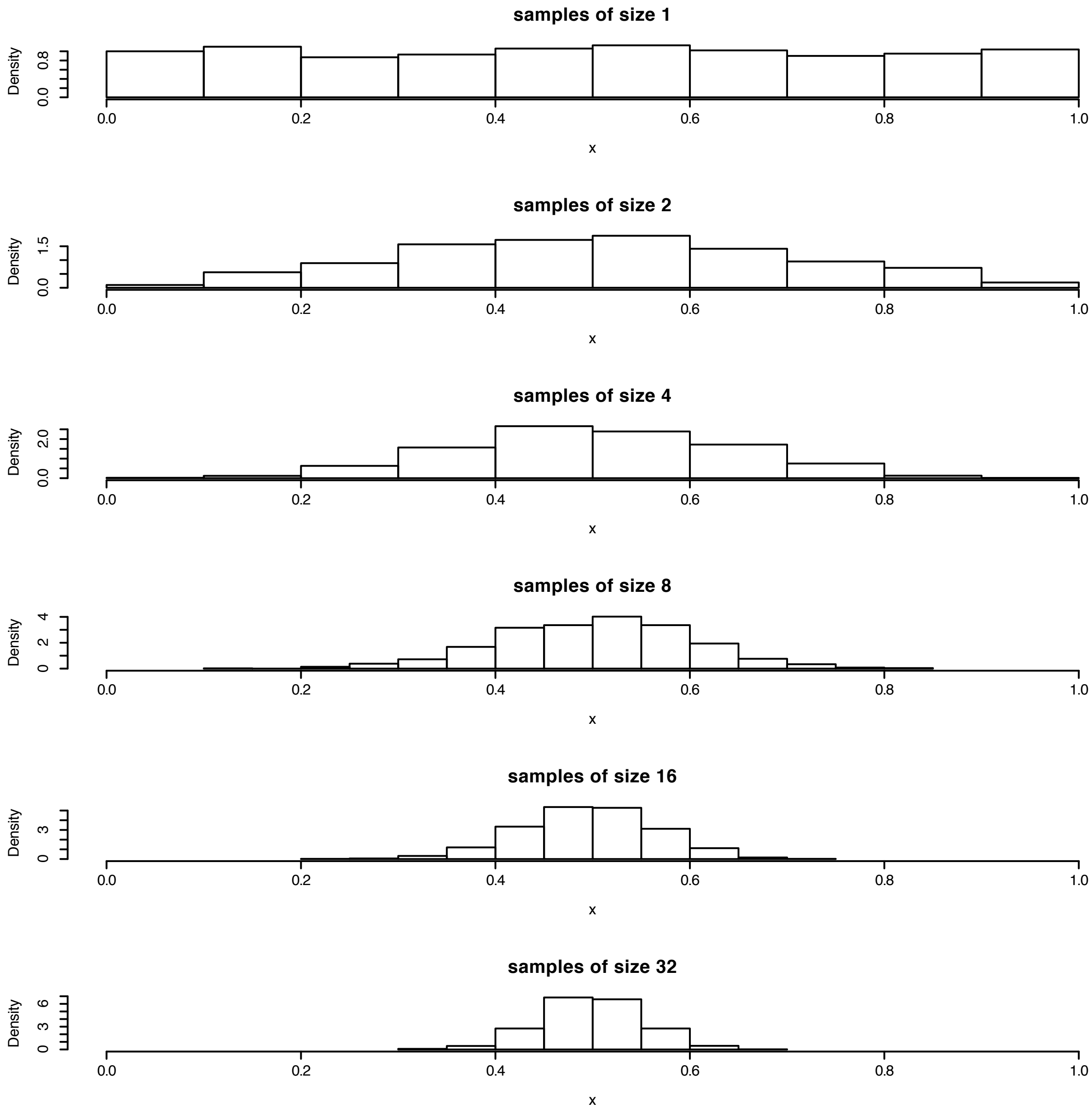
Data = Model + Error

- Almost all of statistics can be summarized as finding how well a model fits the data.
- We need to specify a model, observe the phenomenon, and see how far off the model is from the data.
- Always ask: What is the model? How well does it fit? What are the alternatives? How well do they fit?

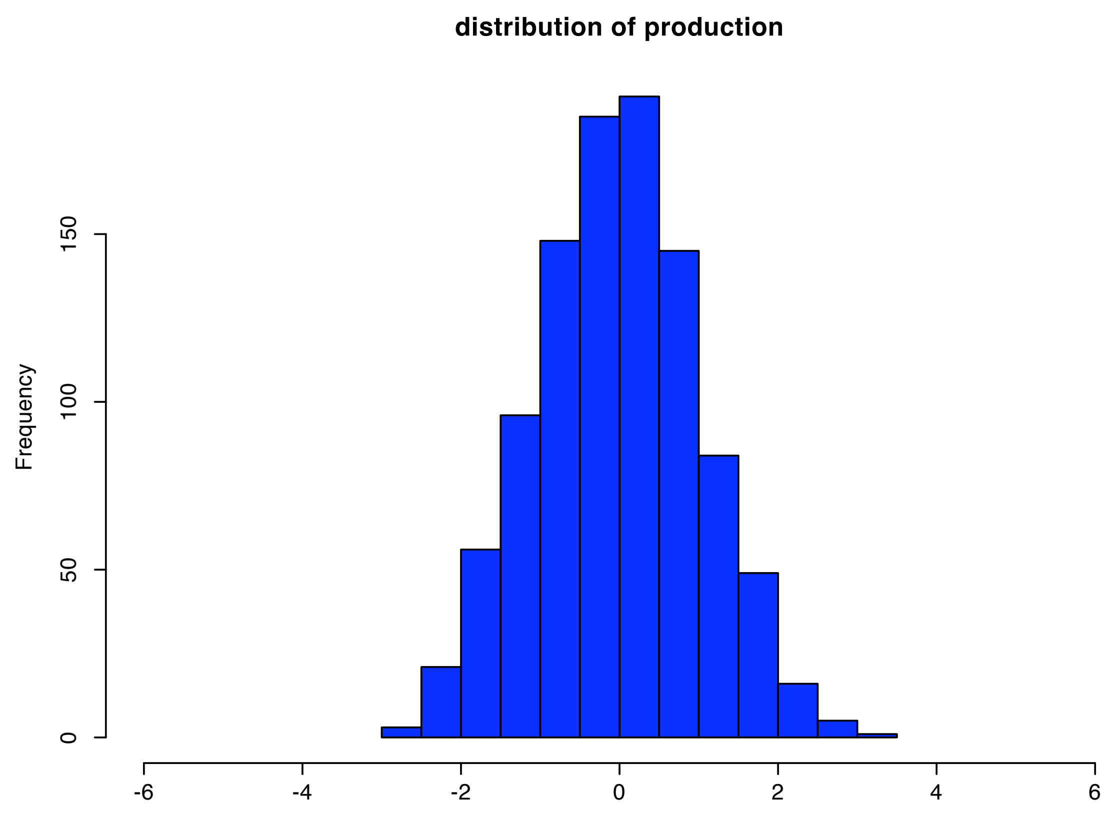
Normal distributions and the central limit theorem

- The distribution of sample means from a population with mean μ and variance σ^2 will tend towards a normal distribution with mean μ and variance σ^2/n

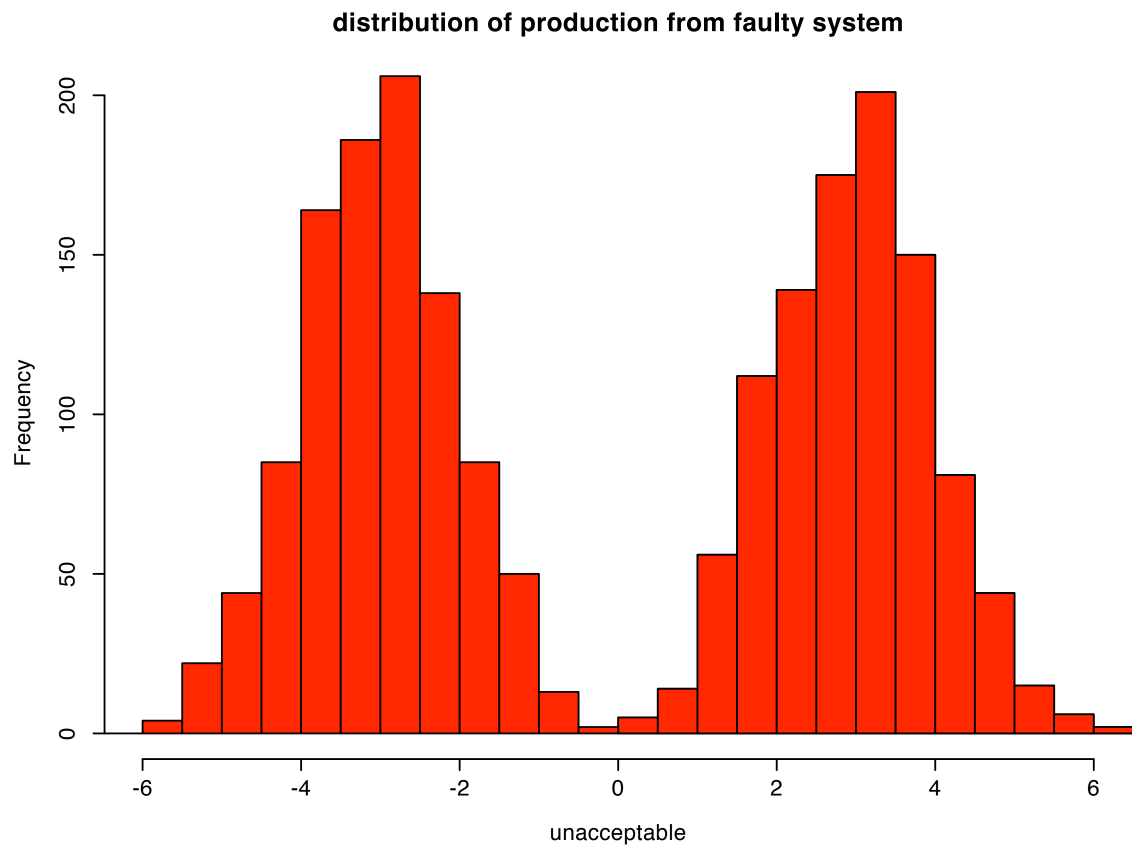
Distributions as f(sample size)



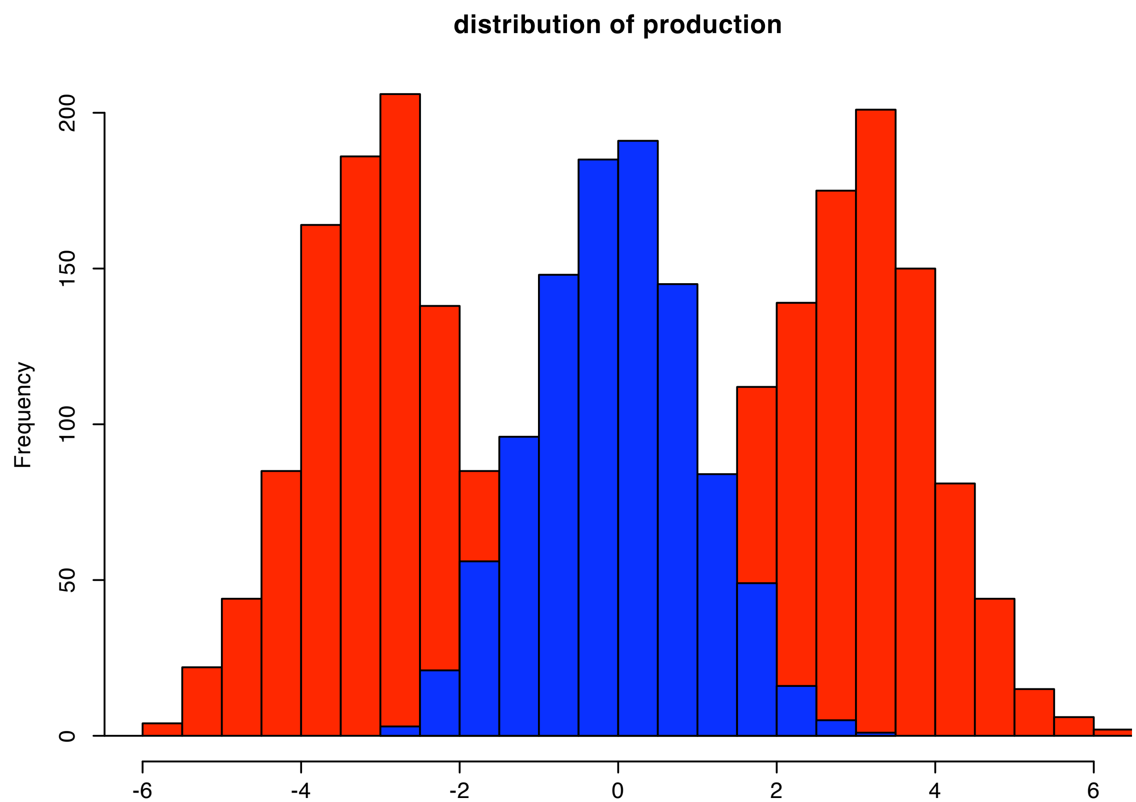
Basic Production process with mean = μ= 0 and σ = 1



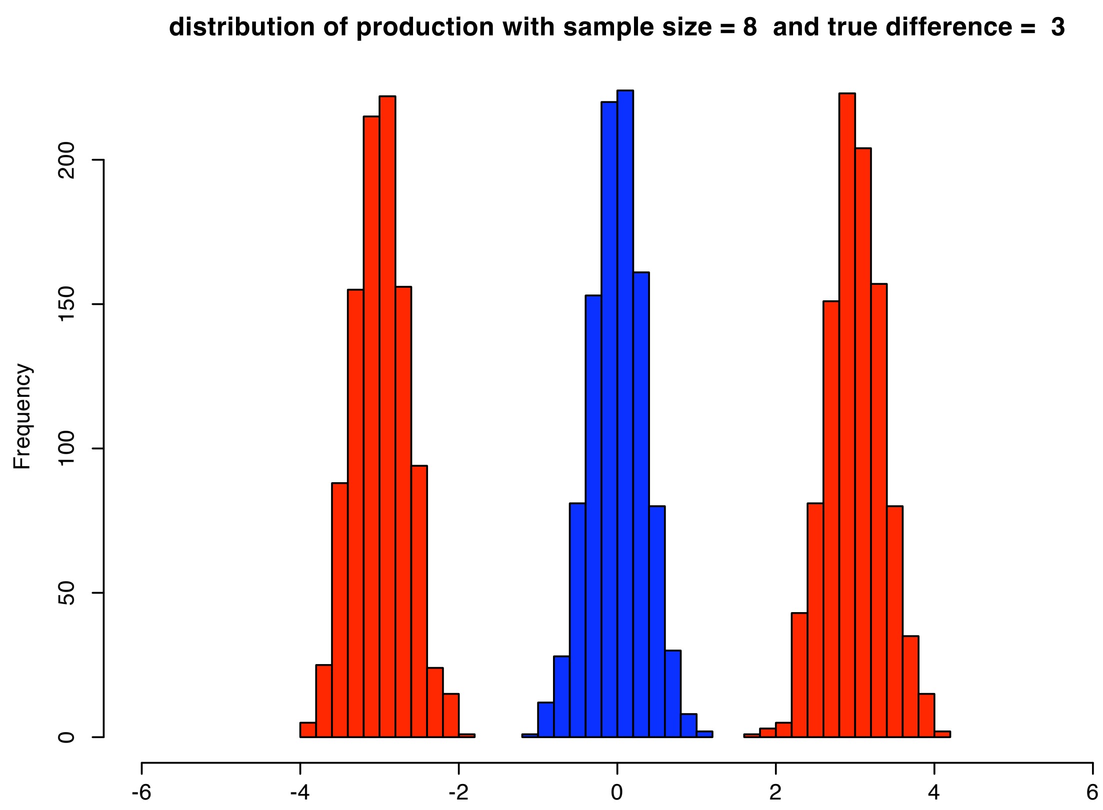
Production can go bad



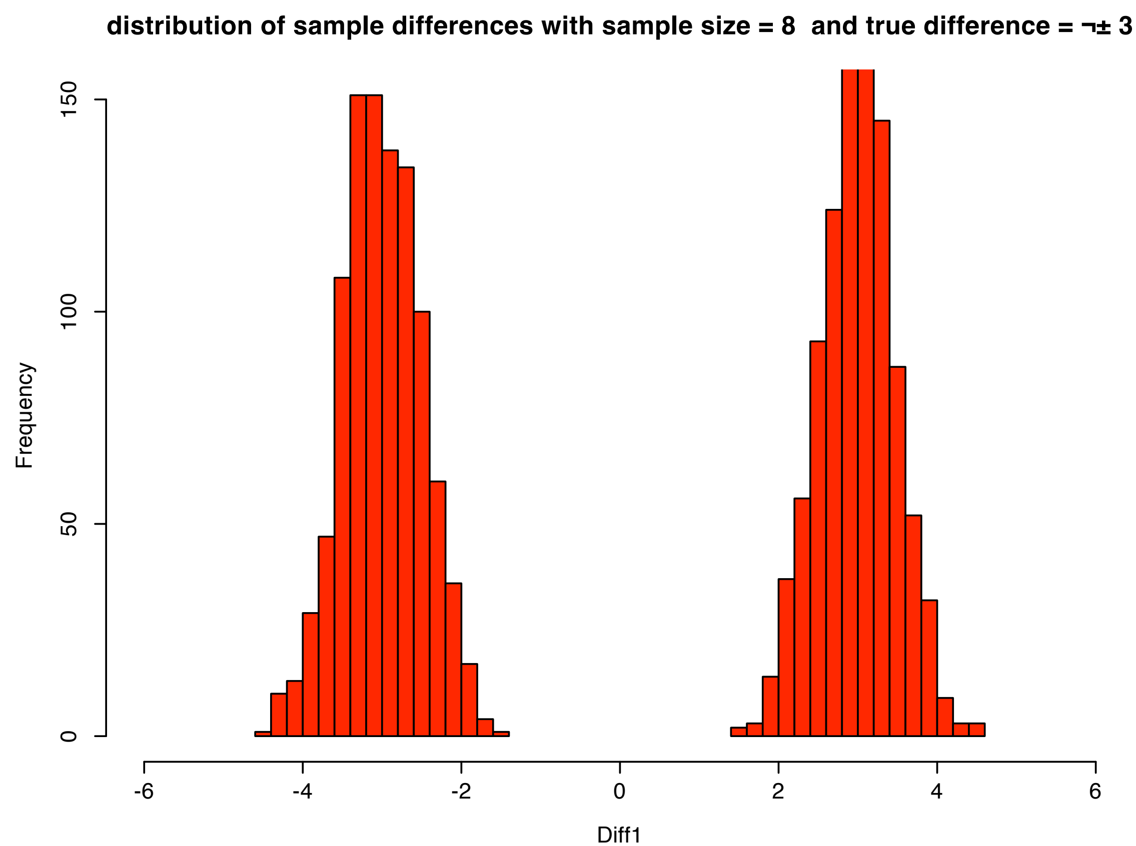
Problem of estimating which state the brewery is in



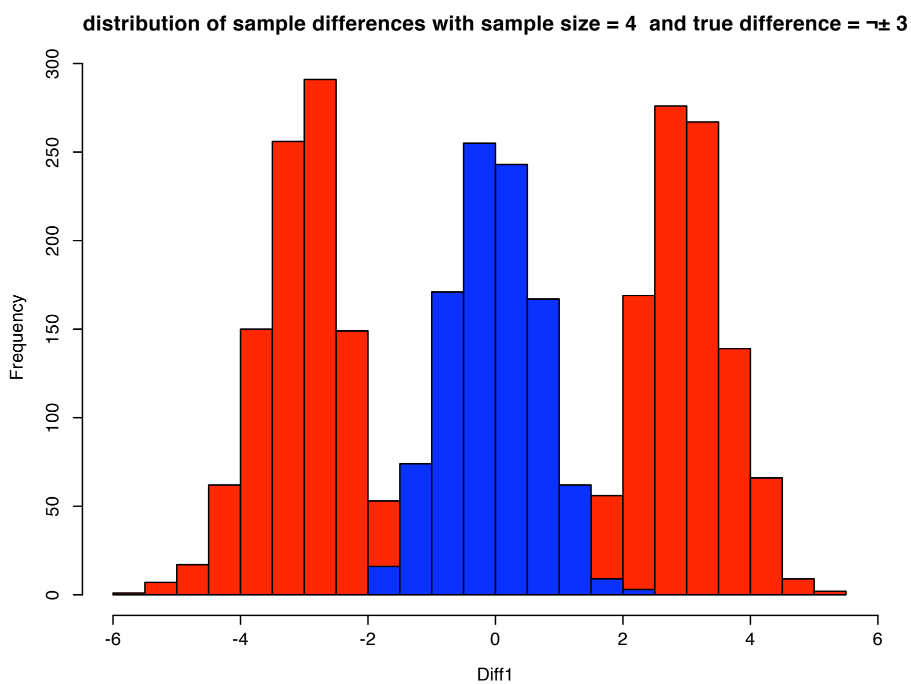
Consider samples of size n

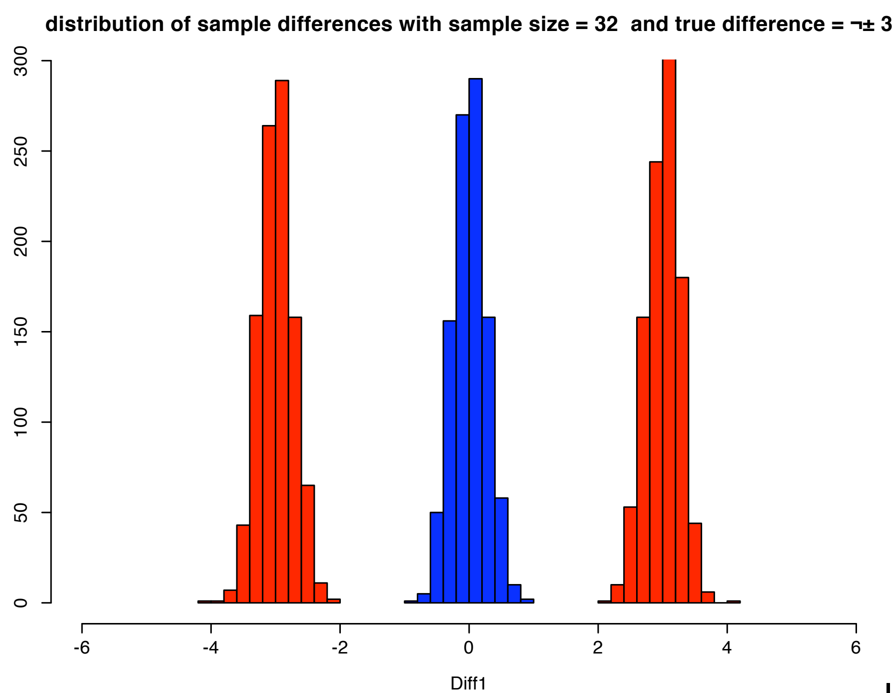


Consider distribution of sample differences

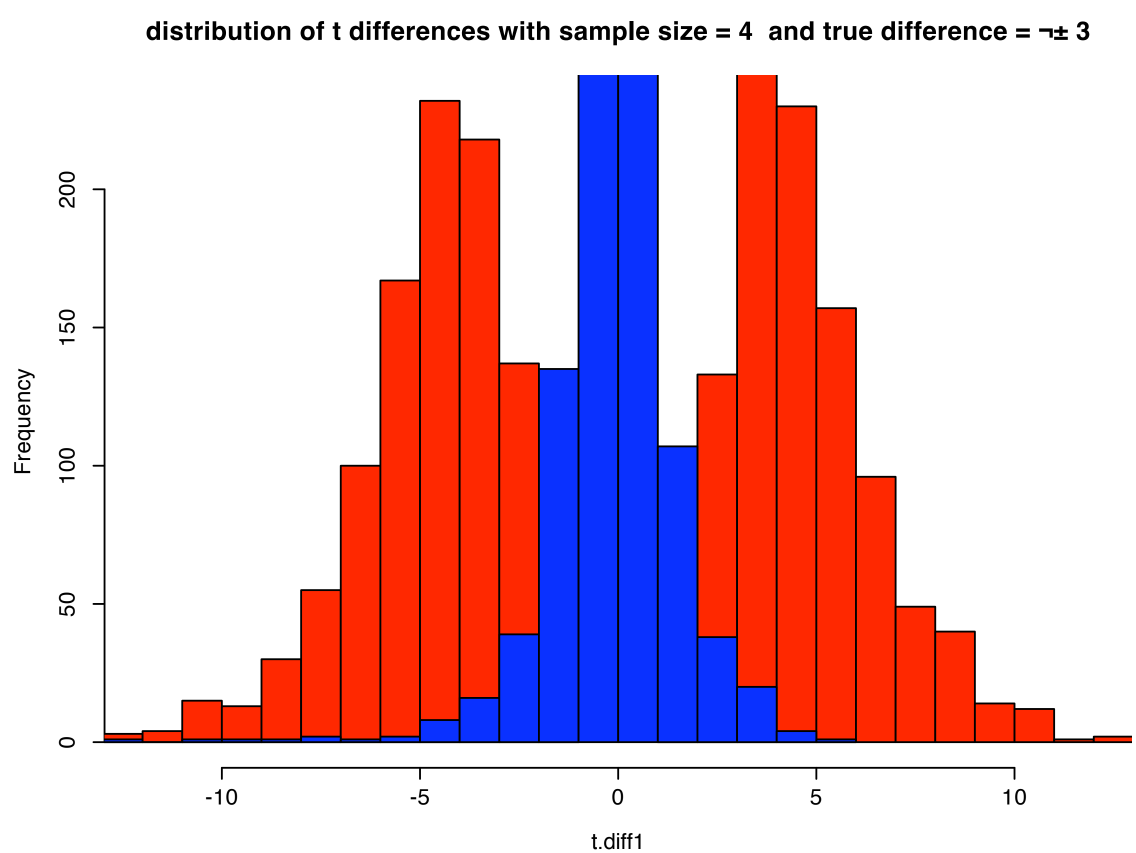


Variation of group differences depend upon sample size

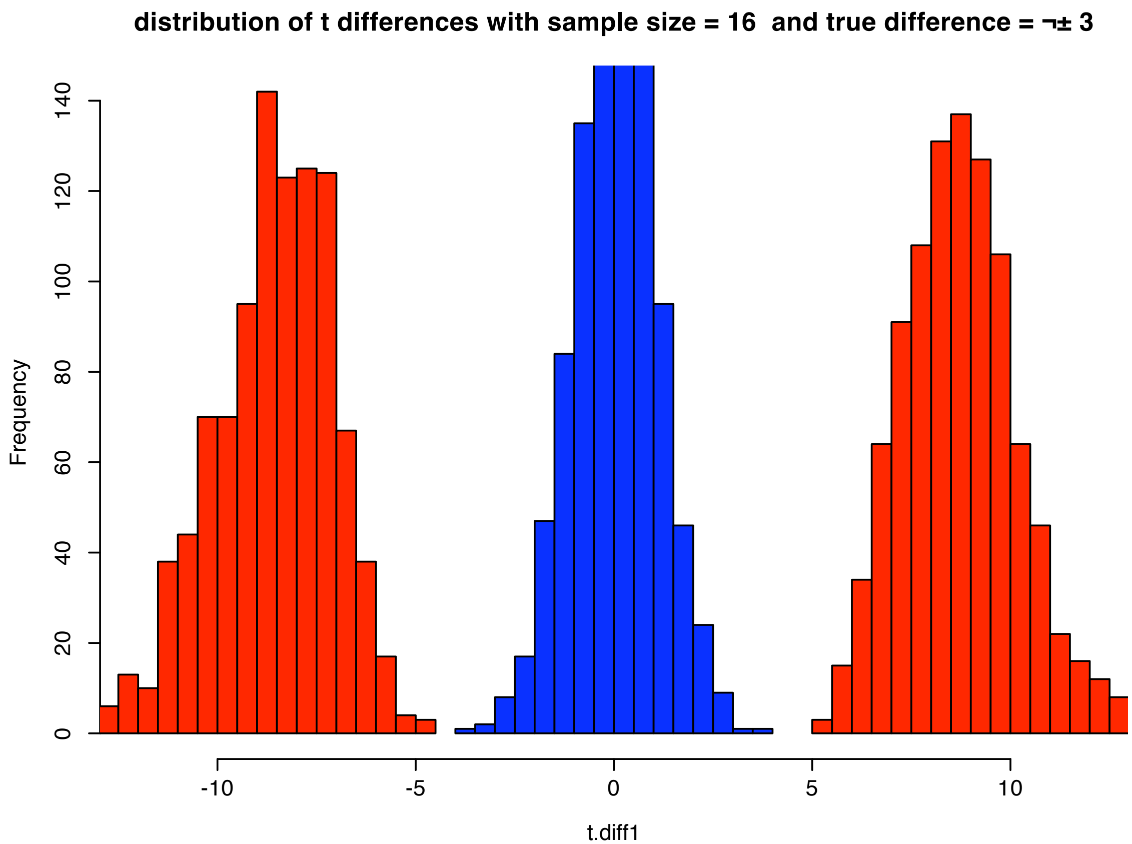




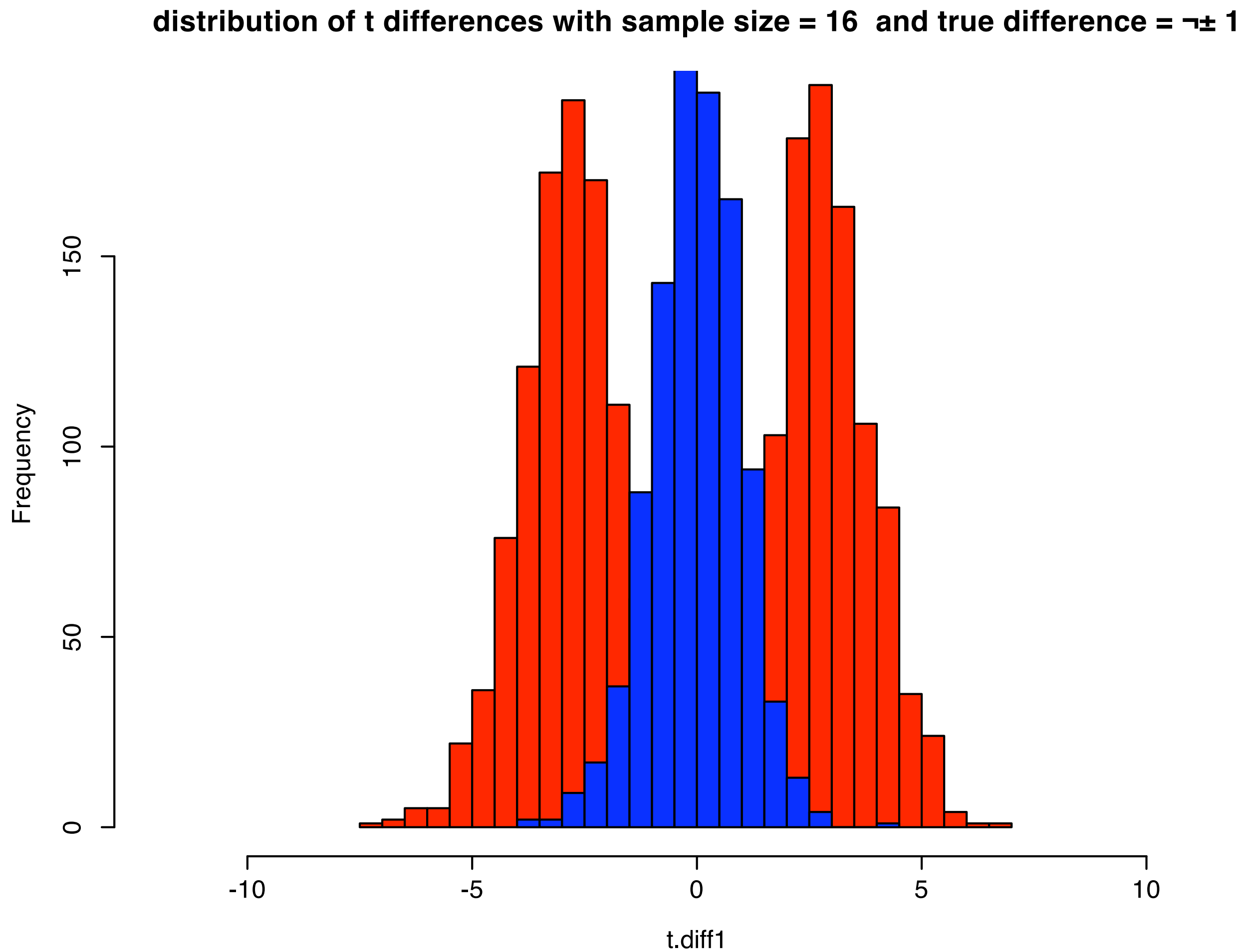
Gossett and the t-test
(compare differences of means to standard error of mean)



A sample of 16, diff = 3



Types of inferential errors
failure to detect, failure to reject



Descriptions and inference

- Classical "Null Hypothesis Inference Statistical Test" NHIST
- Descriptive statistics with confidence intervals
  - expressed in units of measurement
  - expressed in "effect sizes"

Null Hypothesis Testing

- The Null or Nill hypothesis of no difference
- Alternative hypothesis is that Null is wrong
- What is the likelihood of observing differences this big or bigger if Null is true
- If likelihood given Null is small, then reject Null
- Error of false rejection when Null is True (Type I)
- Error of failure to reject when Null is false (Type II)

Critique of NHIST

- Null is never true
- It is not that something has an effect, but we want to know how big the effect is.
- Hookes Law is not that if you pull on a wire it gets longer but rather that the amount it stretches is proportional to the force.
- We need to estimate quantities, not just see if they are $\neq 0$

Descriptive with confidence

- Standard error = $s/\sqrt{N}$
  - observed standard deviation/sqrt(sample size)
- Report observed mean and the standard error of the mean. Allows us to estimate the precision of the estimate.
- If population mean is X, then 68% of observed means will be within 1 se of X, 95% within 2 se of X

Effect size comparisons

- Effect (e.g., difference of means) depends upon the scale we use (meters, feet, inches)
- Standardized effect = effect/within group standard deviation
- Note that while t = effect/sqrt(n) and thus varies as a function of sample size,standardized effect size does not depend upon sample size and thus allows one to compare effects across studies